A Plea for Inexact Truthmaking

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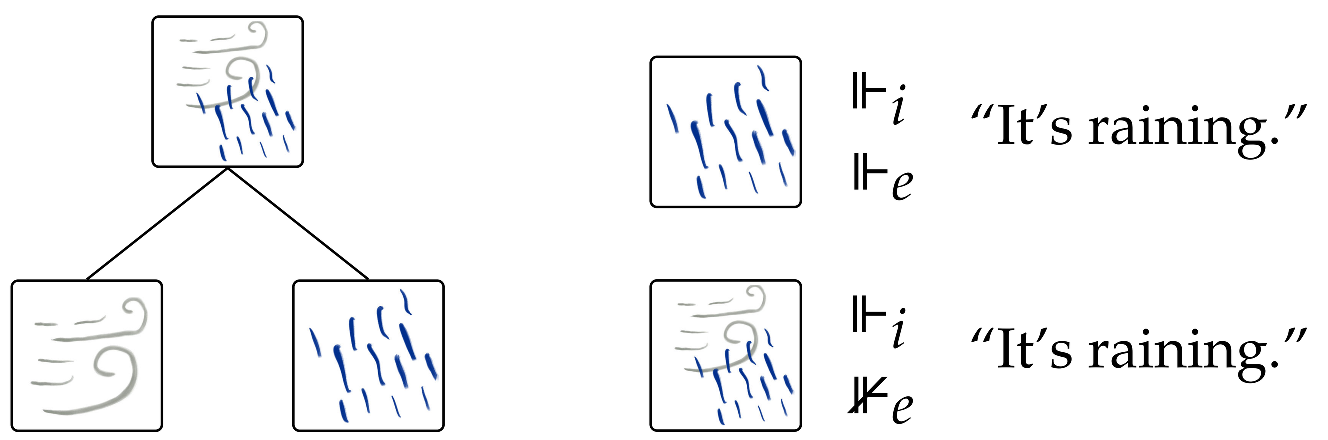
Truthmakers: Exact & Inexact

Fine (2017): Meanings as sets of truthmakers in a state space $\langle S, \sqsubseteq \rangle$, where $\sqsubseteq$ is a partial order on S expressing a part-whole relation, with least upper bounds or fusions ($\sqcup$) defined for all subsets of S.

> Inexact truthmaking: $\mathbf{s} \Vdash_i A$ iff s guarantees A's truth.
>
> Exact truthmaking: $\mathbf{s} \Vdash_e A$ iff s guarantees A's truth *and* *s contains nothing irrelevant to A's truth.*

(1)



Given exact truthmakers atomic sentences (r), we can recursively define $\Vdash_e$ for complex sentences (p, q):

(i) $\mathbf{s} \Vdash_e r$ iff $\mathbf{s}$ is an exact truthmaker of r
(ii) $\mathbf{s} \Vdash_e p \wedge q$ iff $\mathbf{t} \Vdash_e p, \mathbf{u} \Vdash_e q$, and $\mathbf{s} = \mathbf{t} \sqcup \mathbf{u}$
(iii) $\mathbf{s} \Vdash_e p \vee q$ iff $\mathbf{s} \Vdash_e p$ or $\mathbf{s} \Vdash_e q$

Having exact truthmaking is useful for drawing distinctions between logical equivalents which inexact truthmakers do not distinguish:

	A	$A \vee (A \wedge B)$
$\mathbf{a}$	✓	✓
$\mathbf{a} \sqcup \mathbf{b}$		✓

Exact Truthmakers

	A	$A \vee (A \wedge B)$
$\mathbf{a}$	✓	✓
$\mathbf{a} \sqcup \mathbf{b}$	✓	✓

Inexact Truthmakers

We want to use both $\Vdash_e$ and $\Vdash_i$, but

can we define either in terms of the other?

If so, we'd need only one primitive semantic relation.

> **Fine's claim**: we can define $\Vdash_i$ from $\Vdash_e$ but not vice versa.
> I. Exact truthmaking can't be defined in terms of inexact.
> II. We can define inexact truthmaking: $\mathbf{s} \Vdash_i A =_{df} \mathbf{t} \sqsubseteq \mathbf{s}$ and $\mathbf{t} \Vdash_e A$.

My claim: This gets things backwards.

How to Define Exact Truthmaking

A natural approach is to try to define $\Vdash_e$ by using *minimal* inexact truthmakers, which should keep out irrelevant parts.

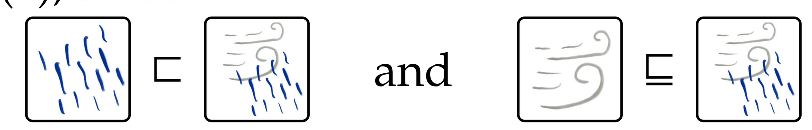
(cf. Berman (1987), Heim (1990), Kratzer (1990, 2002))

s is p-minimal $=_{df}$ $(s \Vdash_i p) \wedge \forall s'(s' \sqsubset s \supset s' \nVdash_i p)$.
s exemplifies p $=_{df}$ s is p-minimal $\vee \forall s'(s' \sqsubseteq s \supset s' \Vdash_i p)$.

We might try the following definition:

$s \Vdash_e p$ $=_{df}$ s exemplifies p.

This works for (1), since



But Fine show this won't work for all cases. We still can't get the distinction between A and $A \vee (A \wedge B)$:

	A	$A \vee (A \wedge B)$
$\mathbf{a}$	✓	✓
$\mathbf{a} \sqcup \mathbf{b}$		

Exemplifiers

Fine concludes that approaches defining $\Vdash_e$ using exemplification are

"doomed to failure" .

But with a cheap trick we can avoid all such problems:

> **Proposal**: use exemplification to define exact truthmakers *for atomic sentences* only, then mimic the recursive characterization of exact truthmakers for complex sentences.
>
> (d.i) $\mathbf{s} \Vdash_e r$ $=_{df}$ $\mathbf{s}$ exemplifies r
> (d.ii) $\mathbf{s} \Vdash_e p \wedge q$ $=_{df}$ $\mathbf{t} \Vdash_e p, \mathbf{u} \Vdash_e q$, and $\mathbf{s} = \mathbf{t} \sqcup \mathbf{u}$
> (d.iii) $\mathbf{s} \Vdash_e p \vee q$ $=_{df}$ $\mathbf{s} \Vdash_e p$ or $\mathbf{s} \Vdash_e q$

This works for $A \vee (A \wedge B)$. More generally, as long as it gets atomic sentences right, it will get the rest right too.

One well known problematic propositionally atomic case (Kratzer 1990, Armstrong 2004):

(2) There are infinitely many stars.

On natural assumptions, there will be no exemplifiers, but there will be exact truthmakers.

Solution: enrich the language, then reapply the strategy.

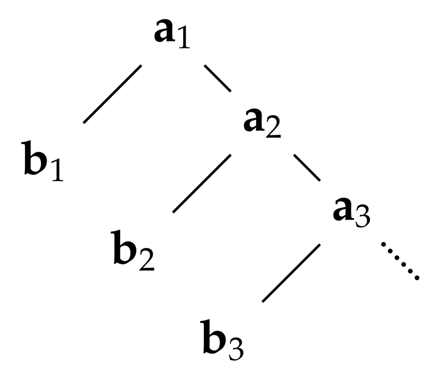
Use exemplification for atomic, unquantified sentences, like 'a is a star.', then mimic the exact truthmaker semantics for quantification.

How *Not* to Define Inexact Truthmaking

According to Fine's definition, a state is an inexact truthmaker of a sentence iff it has an exact truthmaker of the sentence as a part.

> So if we can find cases where there are inexact truthmakers but no exact truthmakers, this definition fails.
>
> I present two such cases below.

Case 1: Weird Mixture



Suppose there is a mixture of a-stuff and b-stuff where every bit of a-stuff contains some (pure) b-stuff.

Now consider the sentence

(3) There is some a-stuff.

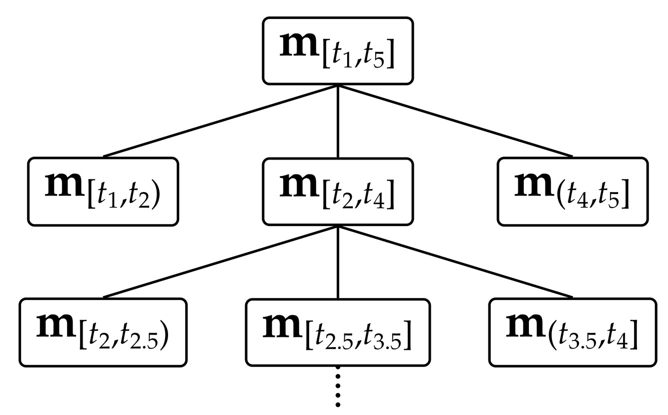
Assuming we have a state space which matches this structure, every inexact truthmaker has a part irrelevant to (3). So,

Inexact truthmakers: $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots$ Exact truthmakers: None

Case 2: Activity & Achievement

Suppose that (i) time is dense, (ii) exact truthmakers for activity sentences like (4-a) must involve proper intervals, and (iii) exact truthmakers for achievements sentences like (4-b) are about points of time.

(4) a. Achilles was moving.
b. The Tortoise won the race.



What are the truthmakers for (5)?

(5) Achilles was moving when the Tortoise won the race.

Inexact: $\mathbf{m}_{[t_1,t_5]} \sqcup \mathbf{w}_{t_3}$, $\mathbf{m}_{[t_2,t_4]} \sqcup \mathbf{w}_{t_3}$, $\mathbf{m}_{[t_{2.5},t_{3.5}]} \sqcup \mathbf{w}_{t_3}$, ...

Exact: None

> **Conclusion**: Fine is right that we can do truthmaker semantics with one primitive semantic relation, but it should be $\Vdash_i$ rather than $\Vdash_e$.